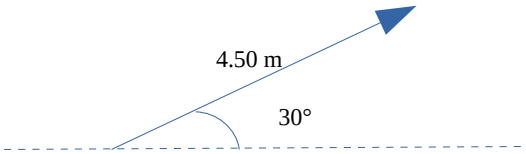
Vectors for Physics

In Physics, we need to be able to use vectors to represent quantities that have both magnitude and direction. For individual vectors, we are going to need to be able to decompose vectors into their component parts, or combine the components to describe the whole vector. For multiple vectors, we are going to need to be able to add and subtract vectors both visually and algebraically. At this point, we do not need to worry about multiplying two vectors together.

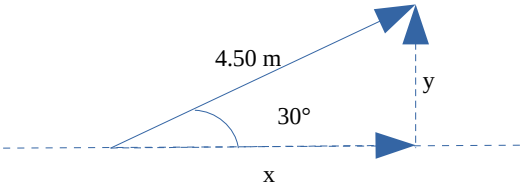
<u>Finding the Components given Hypotenuse and Angle</u>

If given the displacement vector 4.50 meters @ 30° north of east. We would like to be able to know the component of the vector that represent the motion in the east direction and in the north direction.

1. Sketch the vector, labelling the angle and magnitude.



2. Draw the components of the vector. The components are the parts of the vector that are along the axes of the coordinate system you are using. Draw the components as dashed vectors to distinguish them from the actual vector.



3. The trigonometric functions **sine**, **cosine** and **tangent** describe the relationships between lengths and angles in right triangles. For our purposes, we will define the sides of the triangle based on their location relative to the angle given. The **hypotenuse** is the long side of the triangle, always directly across from the right angle. The **adjacent** leg is the is the side of the triangle that touches the angle and the right angle. The **opposite** leg is the side of the triangle that is through the triangle from the given angle. The **sine** function relates the angle given to the ratio of the opposite leg to the hypotenuse. The **cosine** function relates the angle given to the ratio of the adjacent leg to the hypotenuse. The **tangent** function relates the angle given to the ratio of the opposite leg to the adjacent leg. These relationships can be summarized with the following equations:

sin Θ = (opp/hyp) **cos Θ = (adj/hyp)** **tan Θ = (opp/adj)**

In order to find the x-component when given the angle and the hypotenuse, use the cosine function.

cos(30°) = (x/4.50 m) → x = (4.50 m) cos(30°) → x = 3.90 m

In order to find the the y-component, use the sine function.

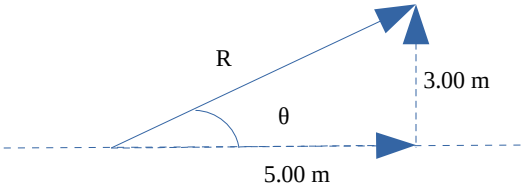
sin(30º) = (y/4.50 m) → y = (4.50 m) sin(30º) → y = 2.25 m

The sign of each component will be determined by the coordinate system you chose. In this case, both are positive.

<u>Finding the Magnitude and Direction given Components</u>

If given that the motion of an object can be described with components of 5.00 m in the x direction, and 3.00 m in the y direction. We would like to know the magnitude and direction of the vector.

1. Sketch the components tip to tail, forming the legs of a right triangle. The vector is represented by the hypotenuse



and the angle between where the tail of a leg connects to the tail of the hypotenuse.
2. Since this is a right triangle, the Pythagorean theorem can be used to find the hypotenuse.

$x^2 + y^2 = Hyp^2 \rightarrow$ $(5.00\ m)^2 + (3.00\ m)^2 = R^2 \rightarrow$ $R^2 = 34\ m^2 \rightarrow R = 5.83\ m$

The tangent function relates the angle to the legs given.

$\tan(\theta) = (y/x) \rightarrow$ $\tan(\theta) = (3.00\ m / 5.00m) \rightarrow$ $\theta = \tan^{-1}(3.00/5.00) \rightarrow$ $\theta = 31.0^o$

Putting the results together, R = 5.83 m @ 31.0º +y of +x.

<u>Practice</u>

Find the components of the following vectors.

1. 5.00 m @ 25º N of E
2. 4.00 m @ 65º N of W
3. 10.0 m/s @ 15º below the horizontal
4. 25.0 N @ 20º W of S
5. 18.5 kg m/s @ 10º -x of +y
6. 49 N @ 40º above the horizontal

Given the following components, find the magnitude-direction form of each vector.

7. 20.0 m N and 30.0 m E
8. 5.00 m/s right and 7.50 m/s down
9. 45.0 N East and 50.0 N South
10. 10.0 kg m/s in -x and 12.0 kg m/s in +y