***Simple Harmonic Motion***

***Concept Development***

Let’s start with the case of a block, mass *m*, sitting on a frictionless surface. We attach the block to a spring with a spring constant *k*, as shown, and have the system sit at rest at the equilibrium point.

1. Explain what is meant by mass, spring constant and equilibrium.

We are going to impose a coordinate system that has zero at equilibrium positive values are to the right. Suppose the block is pulled a distance *A* to the right of equilibrium and released from rest.

2. Draw the force diagrams for the block at the positions shown.

a. –A b. –A/2 c. 0 d. +A/2 e.+A

3. Set up Newton’s Second Law for the block. Solve the equation for acceleration. Summarize the relationship between the position of the block and the acceleration. Include magnitude and direction.

4. What is the relationship between the position of the block and the velocity? Include both magnitude and direction.

The mathematics to take the next step in finding a mathematical set of solutions is beyond the scope of this class. The solution set for describing a motion where $a=-ω^{2}x$ is:

 $x=x\_{0} cos⁡(ωt+φ)$

 $v= -x\_{0}ω sin⁡(ωt+φ)$

 $a= -x\_{0}ω^{2}cos⁡(ωt+φ)$

Any motion that fits this description is called simple harmonic motion. When using trigonometric functions to describe cyclical motion, it is convention to use radians instead of degrees.

5. What do the values of x0, ω, t, ϕ, x0ω and x0ω2 represent (including units)?

6. Sketch the graphs *x vs. t, v vs. t,* and *a vs. t*.

7. What is ω for the case of the spring-block combination on a frictionless surface?

8. The period is the time it takes for one complete cycle of motion. Looking at the equations for motion, what is the condition for one period to pass?

9. How does the mass of the block impact the period of the system?

10. How does the spring constant impact the period of the system?

11. The frequency of an object represents the number of cycles of motion per unit time. What equation relates period and frequency?

12. Summarize the equations for the period and frequency of a spring-mass system.

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***Exercises:***

1. A block is attached to a spring and oscillates with a period of 0.20 seconds when pulled 15 cm from equilibrium and released from rest.

 a. What is the frequency of the motion?

 b. What would happen to the period if the amplitude increased to 30 cm?

2. A 5.0 kg block is attached to a spring with k = 500. N/m and placed on a horizontal frictionless surface. The block is pulled 10.0 cm from equilibrium and released from rest.

 a. What is the maximum acceleration of the block and where does it occur?

 b. What is the maximum velocity and where does it occur?

 c. What is the period of the motion?

 d. How far does the block travel in one period?

3. A 0.50 kg block is oscillating at the end of a spring with a frequency of 2.0 Hz. What is the spring constant?

4. Prove that a vertical spring-block system behaves in a similar fashion to the horizontal case. Show that the motion is simple harmonic about the equilibrium point.

5. A mass at the end of a spring is stretched 8.0 cm from equilibrium and released. At what distance from equilibrium will it have:

a. velocity equal to half its maximum velocity?

 b. acceleration equal to half its maximum acceleration?

6. A 2.0 kg block slides along a horizontal surface at 4.0 m/s before it hits a spring with k = 800 N/m.

 a. How far is the spring compressed at its maximum?

 b. How long will it take for the block to return to the equilibrium point?

7. Prove that a simple pendulum with a mass *m* and length *l* exhibits simple harmonic motion with $ω= \sqrt{\frac{g}{l}}$.

8. What is the length of a pendulum whose period is 0.50 seconds on Earth?

9. On a different planet, the period of a 0.75 meter pendulum is 2.0 seconds. What is the value for *g* on this planet?

10. What is a reasonable estimate of the period of a swing on an elementary school playground? Show calculations and explain your assumptions.