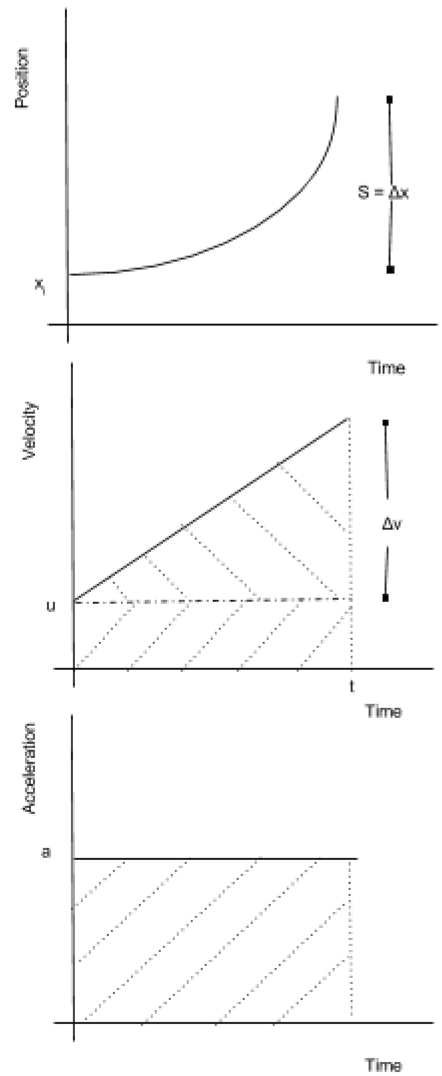
Derivation of SUVAT Equations



In order to derive the equations of motion for an object from the graphs, there are a couple of assumptions that need to be addressed.

- Acceleration is constant along the period of motion. This is required for the particular results here to be valid. This does not have to be true in the real world, but a careful analysis would then require an understanding of Calculus and is beyond the scope of this course.
- Acceleration, initial velocity and initial position will be shown as positives. These are not requirements for the applications of the results. This is chosen for simplicity of representation.

Acceleration to Velocity

Acceleration is defined as the change in the velocity of an object with respect to time. The area under the acceleration graph will give the change in the velocity over that time segment. The assumption that the acceleration is a constant leads the graph to be a rectangle with height *a* and length *t* so $\Delta v = at$.

Now that we know the change in the velocity, we add that to the initial velocity, *u*. So, $v = u + at$, is a straight line beginning at *u* with a slope of *a*.

Velocity to Position

Δ = change (final - initial)
s = displacement
u = initial velocity
v = final velocity
a = acceleration
t = time

Velocity is the change in the position with respect to time. The area under the velocity graph will give the change in the position for that time segment. Since the velocity is not a constant value, it is easier to break the area into a rectangular section and a triangular section. The height of the rectangular section is *u*, the initial velocity. The height of the triangular section is Δv. This gives the change in the position to be $\Delta x = ut + \frac{1}{2}\Delta v t$. From the previous analysis, $\Delta v = at$. Therefore $\Delta x = ut + \frac{1}{2}(a\,t)t$. If the goal is to find the displacement (*s*) of the object, we can stop with $s = u\,t + \frac{1}{2}at^2$, which is the equation given in the IB Data Booklet. If the goal is to complete the graph, we can add to the initial position and get the equation $x_f = x_i + ut + \frac{1}{2}at^2$. The graph is therefore a parabola with the shape determined by the initial velocity and the acceleration.

Shortcut Equations

The previous equations are built directly from the defined quantities. All constant acceleration problems can be solved using those equations as long as only two quantities are unknown. If acceleration and time are both unknown, the solution would necessarily include solving a system of two equations with two unknowns. While this is mathematically fine, the point of the course is to work with Physics concepts, not to test mathematical ability.

If we don't know time and aren't trying to calculate that value, we can perform a mathematical substitution and solve.

Solving $v = u + at$ for time gives $t = \frac{v-u}{a}$.

Substituting into the larger equation:

$$s = u(\tfrac{v-u}{a}) + \tfrac{1}{2}a(\tfrac{v-u}{a})^2$$
$$s = \tfrac{uv-v^2}{a} + \tfrac{v^2-2uv+u^2}{2a}$$
$$2as = 2uv - 2v^2 + v^2 - 2uv + u^2$$
$$v^2 = u^2 + 2as$$

Another equation for the displacement can be derived from the work done in finding the previous displacement equation.

$$s = ut + \tfrac{1}{2}\Delta vt \text{ and } \Delta v = v - u$$
$$s = ut + \tfrac{1}{2}(v-u)t$$
$$s = (\tfrac{v+u}{2})t$$

Data Booklet Equations

The kinematics equations that appear in the IB Physics Data Booklet are:

$$v = u + at$$
$$s = ut + \tfrac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = (\tfrac{v+u}{2})t$$

Remember that these equations are only valid if the acceleration is a constant.