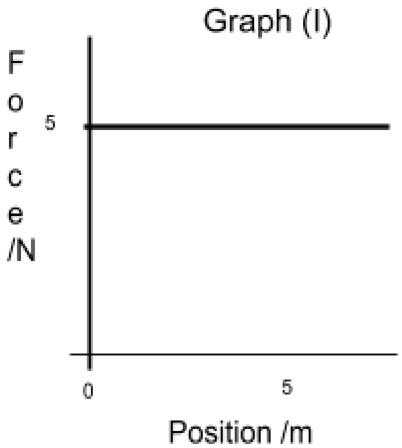
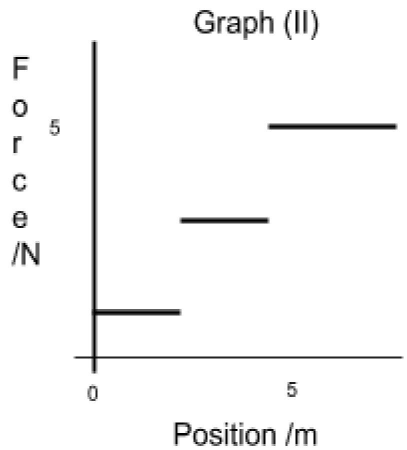
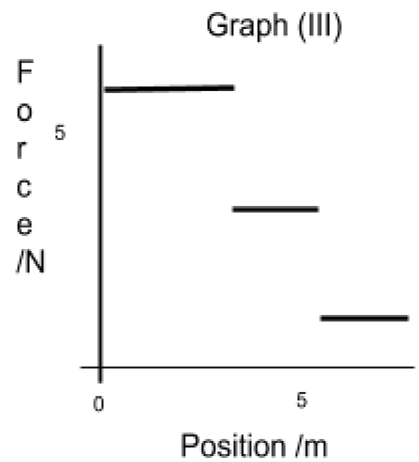
Even More Work - Elastic Energy

1-4. Suppose that a 3.0 kg block is pulled along a horizontal frictionless surface by a horizontal force that is described by the graphs below.







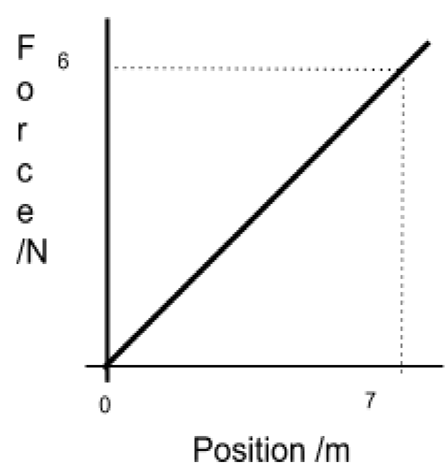
1. In Graph (I), the force is a constant 5.0 Newtons for the entire 7.0 meters.
   a. Calculate the work done by the force during the 7.0 meters.
   b. Determine the final speed at x = 7.0 m if the block was initially at rest at x= 0 m.

2. In Graph (II), the force increases along the motion. The force is 1.0 N for the first 2.0 m, 3.0 N from 2.0 m to 4.0 m and 5.0 N from 4.0 m to 7.0 m.
   a. Calculate the work done by the force from x = 0.0 m to x = 2.0 m.
   b. Calculate the work done by the force from x = 2.0 m to x = 4.0 m.
   c. Calculate the work done by the force from x = 4.0 m to x = 7.0 m.
   d. Show that the total work done from x = 0.0 m to 7.0 m is 23.0 J.

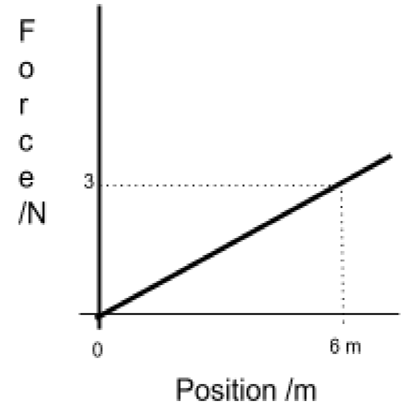
3. In Graph (III), the force decreases along the motion. The force is 6.0 N from 0 m to 3.0 m, 3.0 N from 3.0 m to 5.0 m and 1.0 N from 5.0 m to 7.0 m.
   a. Calculate the work done by the force from x = 0.0 m to x = 7.0 m.
   b. Show that the the speed of the block at 7.0 m is 4.28 m s^{-1} if the block is moving 1.0 m s^{-1} at x = 0.0 m.

4. If the force is a constant value over the section of motion, you can use the equation $W = F \bullet s$. If the force changes along the motion, you must break the motion into constant force sections. An easier way to represent this process is to use the graph. Shade the area on each graph that represents the work done by the force.

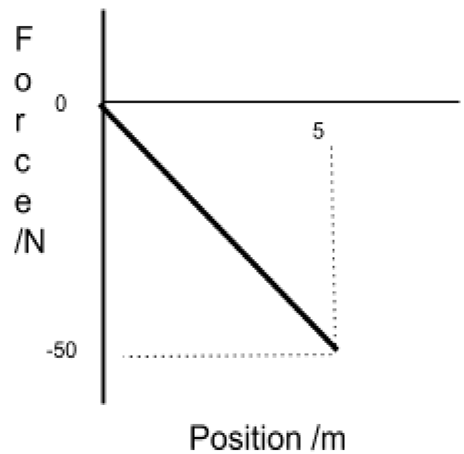
5. A block is pulled by a force that changes continuously with the position as shown on the right. In this case, $W = F \bullet s$ isn't appropriate because the constant force sections are mathematically small. The graphical method is still appropriate however.
   a. Shade the area of the graph that represents the work done by the force from x = 0.0 to x = 7.0 m.
   b. Determine the amount of work done by the force from x = 0.0 m to x = 7.0 m.



6. A 1.0 kg block is pulled along a horizontal, frictionless surface by the varying force that is shown at the right. The block starts at rest at x = 2.0 m.
   a. Shade the area of the graph that represents the work done by the force from x = 2.0 m to x = 6.0 m.
   b. Determine the amount of work done by the force from x = 2.0 m to x = 6.0 m.
   c. Show that the speed of the block is 4.0 m s^{-1} when it is at the position x = 6.0 m.



7. The spring force is a special case of a variable force that fits the definition of a conservative force. The work done by the force is independent of the path taken from one position to the next. This means that if we include the spring in our system, we can define an elastic potential energy by using the relationship $\Delta E_{Potential} = -W$.
   a. The force of a spring is described by the equation $F_{spring} = -kx$. Determine the value for *k* for the spring described by the graph to the right.
   b. Calculate the amount of work done by the spring in moving from x = 0.0 m to x = 5.0 m.
   c. Use the equation $\Delta E_{Potential} = -W$ to calculate the amount of energy stored in the spring at the position x = 5.0 m
   d. Derive an equation for the amount of work that would be done by the spring in moving from x = 0.0 m to any position x.
   e. Using the equation $\Delta E_{Spring} = -W_{Spring}$, derive an expression for the amount of energy that would be stored in the spring at any position x.
   f. Now we will generalize to any spring with spring constant *k*.
      Derive and equation for the amount of work done by the spring in moving from x = 0.0 m to any position x.
   g. Using the equation $\Delta E_{Spring} = -W_{Spring}$, derive an expression for the amount of energy stored in the spring at any position x.



8. A 1.5 kg block is attached to the end of a spring (k = 400 N m^{-1}) and placed on a horizontal surface. The spring is compressed 25 cm and released from rest.
   a. Calculate the potential energy of the spring-block system when the spring is compressed 25 cm.
   b. Determine the amount of kinetic energy the block has when it passes the relaxed point of the spring.
   c. Show that the block is traveling 4.08 m s^{-1} when it passes the relaxed point of the spring.

9. A 4.0 kg block is sliding along a horizontal, frictionless surface at 10 m s^{-1} when it hits a spring that has a spring constant of 500 N m^{-1}.
   a. Calculate the kinetic energy of the block immediately before it hits the spring.
   b. Show that the block travels 0.89 meters before it stops.