Errors and Uncertainty

Every measurement has a degree of uncertainty. This simply means that we are not dealing with perfect numbers in laboratory situations. As a result, we must mitigate errors as much as possible and acknowledge the uncertainty that comes from taking measurements. When discussing data collection, it is helpful to keep in mind the following concepts:

> **Accuracy** - How close a measurement is to the accepted value.
> **Precision** - How close a measurement is to similar measurements.
> **Random Error** - An error due to the behavior of the observer. These typically show up due to difficulties reading instruments. The resulting measurements end up on both sides of the accepted value.
> **Systematic Error** - A error due to incorrectly calibrated instruments. The resulting measurements end up being skewed in a single direction from the accepted value.

Mitigating Errors

1. While taking data, take a moment to think about possible sources of errors before you begin.
2. Choose appropriate measurement devices. Check the devices for calibration before taking any data.
3. Be consistent in the methods used in measurement.
4. Take multiple trials.

Uncertainty in Measurements

When taking measurements, keep in mind that every device has an uncertainty. That uncertainty dictates the level of detail for each measurement.

> If we have an *analog* device (meter stick), it is assumed to be accurate to half of the smallest division. As a result, measure to that level of detail and estimate one place beyond the marked value.
>
> If we have a *digital* device (pan balance), it is assumed to be accurate the the smallest digit reported. There is no way to know how to estimate another place value, so we have to accept what is on the meter.

I. Suppose that we have taken a series of measurements and need to report a value to be used in calculations.

We'll use the example of times below:

Trial 1 /s	Trial 2 /s	Trial 3 /s	Trial 4 /s	Trial 5 /s
2.85	3.01	2.79	2.94	3.05

Since we have a range of values, we will let the data guide us to a value for the value with an uncertainty. We are assuming that the times are the result of measuring the same section of the repeated motion, using the same technique and are subject to averaging. So,

$$t_{avg} = \frac{t_1+t_2+t_3+t_4+t_5}{5} = \frac{2.85+3.01+2.79+2.94}{5} = 2.93\ s$$

In order to determine the uncertainty in the value of time, we have an option.

Option 1: Maximum difference from average

1. Find the maximum and minimum values (*3.05 s and 2.79 s)*
2. Find the difference between each of these values and the average (*3.05 - 2.93 = 0.12 s and 2.93 - 2.79 = 0.14 s)*
3. The largest difference from the average is the uncertainty (*0.14 s)*
4. Report the final answer *(2.93 s +/- 0.14 s)*

Option 2: Half the range

1. Find the maximum and the minimum values (*3.05 s and 2.79 s)*
2. Find the range by calculating the difference between the maximum and minimum values *(3.05 - 2.79 = 0.26 s)*
3. The uncertainty is half of the range *(0.26/2 = 0.13 s)*
4. Report the final answer *(2.93 s +/- 0.13 s)*

In both cases, the uncertainty is not allowed to have more specific information than the measurement. This means that if the measurement is valid to the hundredths place, the uncertainty cannot have information about the thousandths place

In some circumstances, it is acceptable to round the measurement and uncertainty so there is only one place of uncertainty (*2.9 s +/- 0.1s).* In some cases, this is considered to be too cautious and unnecessary.

II. Suppose that there is no range in the values that we measure. We can still determine an uncertainty by using the uncertainties given by the type of device.

This would mean one half of the smallest division on the device for an analog device which required an estimation. Since there is no room for estimation for a digital device, assume the smallest digit is uncertain.

It is often helpful to find the **Fractional Uncertainty** of a measurement. This allows us to compare the uncertainty to the measurement. The fractional uncertainty is the ratio of the uncertainty to the value. The fractional uncertainty of the time from above would be:

$$\frac{\Delta t}{t} = \frac{0.13\ s}{2.93\ s} = 0.044$$

Notice that there are no units for the fractional uncertainty of a measurement.

We will use **Percent Uncertainty** more often than fractional uncertainty when examining the impact of uncertainty on a number. The percent uncertainty tells what percent of the value is uncertain. Using the same values from above,

$$\frac{\Delta t}{t} x100\% = \frac{0.13\ s}{2.93\ s} x100\% = 4.4\%$$

There are times when information will be given in terms of percent uncertainty instead of absolute uncertainty

$$t = 2.93\ s\ \pm\ 4.4\%$$

Managing Uncertainty in Calculations

Let's assume that $A = 2.5\ m\ \pm 0.3\ m$ and $B = 3.0\ m\ \pm 0.4\ m$. The meaningful uncertainty of a quantity cannot decrease under calculation.

Adding or Subtracting Values

When adding or subtracting values, add the absolute uncertainties together.

If $C = A \pm B$, then $\Delta C\ =\ \Delta A + \Delta B$

$$\Delta C\ =\ 0.3\ m\ +\ 0.4\ m\ =\ 0.7\ m$$

Multiplying or Dividing Values

When multiplying or dividing values, the fractional uncertainty of the result is the sum of the fractional uncertainties.

If $D = A \cdot B$ or $D = A/B$, then $\frac{\Delta D}{D} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

(Assuming $D = A \cdot B = 7.5\ m^2$ $\frac{\Delta D}{7.5\ m^2} = \frac{0.3\ m}{2.5\ m} + \frac{0.4\ m}{3.0\ m} \Rightarrow \Delta D = 1.9\ m^2$

Powers

If you are raising a value to a power (squaring, cubing, square root, …), multiply the fractional uncertainty by the power.

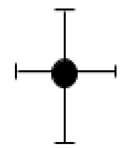
If $E = A^n$, then $\frac{\Delta E}{E}\ =\ n \cdot \frac{\Delta A}{A}$

Suppose $E = A^3$, then $\frac{\Delta E}{27} = 3(\frac{0.3\ m}{3}) \Rightarrow \Delta E = 8\ m^3$.

Determining Uncertainty though Graphing

Plotting Points

- Set up a scale on each axis that allows you to make the graph large enough to read and allows for interpolation. (Using markings to represent factors of 10 multiples of 1, 2 or 5 tends to improve estimation, but is not required.)
- Plot the average values as points, ignoring the uncertainty.
- Add the vertical and horizontal (if present) error values to show the possible spread in what is known.



Best-Fit Line

- DO NOT PLAY CONNECT THE DOTS! That assumes that every data point is perfect and there is no underlying trend.
- We are going to assume that there is no directional bias (systematic error) in the data unless we are given evidence to believe otherwise. With that in mind, we should assume that we have roughly the same number of data points above the “real” pattern as below. Choose a line that demonstrates this tendency.
- Use points on your best-fit line to calculate the slope. Drawing the best-fit line is a way of identifying the underlying pattern. Using data points not on the line indicates a lack of trust in the line.

Uncertainty in the Slope

- Estimate the worst acceptable lines to find the range. One should be the steepest acceptable and the other the shallowest acceptable.
- Find the slope of each of these lines.
- The uncertainty equals half the range of the slopes.

$$\Delta m = \frac{m_{steepest} - m_{shallowest}}{2}$$